

On the Einstein-Podolsky-Rosen paradox using discrete time physics

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1 Abstract

The Einstein-Podolski-Rosen paradox highlights several strange properties of quantum mechanics including the super position of states, the non locality and its limitation to determine an experiment only statistically. Here, this well known paradox is revisited theoretically for a pair of spin $\frac{1}{2}$ systems in a singlet state under the assumption that in classical physics time evolves in discrete time steps Δt while in quantum mechanics the individual spin system(s) evolve(s) between the eigenstates harmonically with a period of $4\Delta t$. It is further assumed that time is a single variable, that the quantum mechanics time evolution and the classical physics discrete time evolution are coherent to each other, and that the precision of the start of the experiment and of the measurement time point are much less than Δt . Under these conditions, it is demonstrated for a spin $\frac{1}{2}$ system that the fast oscillation between the eigen states spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$ reproduces the expected outcome of a single measurement as well as ensemble measurements without the need of postulating a simultaneous superposition of the spin system in its quantum state. When this concept is applied to a spin $\frac{1}{2}$ system pair in a singlet state it is shown that no entanglement between the two spins is necessary to describe the system resolving the Einstein-Podolski-Rosen paradox.

2 Significance Statement

The Einstein-Podolsky-Rosen paradox is one of the most intriguing concepts in physics that brought to attention the very strange phenomena of entanglement in quantum mechanics, which finds experimental support through the Bell inequalities and correspondingly set up experiments. However, by introducing time as a single variable, which is continuous in quantum mechanics but discrete in classical physics, time becomes a non local hidden variable that resolves both the Bell inequalities and the Einstein-Podolsky Rosen paradox as demonstrated here for a two spin $\frac{1}{2}$ system in a singlet state because no quantum entanglement is required to describe the singlet state.

3 Introduction

The Einstein-Podolsky-Rosen paradox (EPR) (1) is a stimulating Gedankenexperiment (thought experiment) in quantum mechanics challenging quantum mechanics as a complete description of physical reality. Motivated by the EPR paradox Schrödinger introduced the term entanglement (in german: Verschränkung) (2). An entangled system is thereby defined as a system whose quantum state cannot be described by a product of states of its individual constituents. With other words, if a system is entangled, the state of one constituent is not independent from the other and thus can not be fully described without considering its counterpart. A prototypical example thereof is the singlet state of two particles with spin

$\frac{1}{2}\hbar$ (normalized in the following) having together spin zero (3). Because the total spin is zero, whenever the first particle is measured to have spin up on some axis i $\langle \uparrow_1 | S_i | \uparrow_1 \rangle \equiv \langle \uparrow_1 | \uparrow_1 \rangle_i$, the other (when measured on the same axis) is always observed to have spin down $\langle \downarrow_2 | \downarrow_2 \rangle_i$ even upon long distance separation. Vice versa, if the first particle is measured to have spin down, the other will have/has spin up, respectively. It is the paradox that before the measurement particle 1 is superimposed and thus no decision has been made yet whether at the measurement it will have spin up or down, but ones the measurement is performed the state of the entire entangled system collapses instantaneously so that particle 2 has the orthogonal spin state of particle 1 without requiring any information transfer (which requires time) between the particles. Because such a behavior is regarded to violate causality, Einstein and others concluded that the established formulation of quantum mechanics must be incomplete (1). A possible solution to this dilemma appeared to be the introduction of hidden variables, which, while not accessible to the observer, determine the future outcome of the spin measurements before the separation of the two particles. With other words, each particle carries the necessary information with it upon separation, and thus no transmission of information transfer from one particle to the other is required at the time of measurement. However, Bell's inequality theorem [4] rules out local hidden variables as an explanation of the mentioned paradox and since the Bell inequalities have been supported by experiments (5-7), EPR is considered no longer to be a paradox and the non local nature of quantum mechanics is thus widely accepted. It is however important to note that Bell's theorem still permits the existence of non-local hidden variables such as the Bohm interpretation of quantum mechanics (8), which states that all particles in the universe are able to exchange information instantaneously, or less demanding that there is a non-local connection of some sort between the system under study and the measurement devices as well as between the measurement apparatus themselves.

In the presented approach the latter concept is realized by the assumption, that time (unlike space coordinates) is a single variable and that time is continuous in quantum mechanics but in classical physics evolves in discrete steps in a coherent fashion between the two frames. This rational guarantees that both measurement devices as well as the system under study are in clock and thus non-locally connected. Furthermore, it is assumed that in the quantum mechanical description of the system there is no superposition of quantum states, but rather a fast oscillation with a periodicity proportional to the classical time step size. This modification to quantum mechanics allows not only to describe a quantum system without super position, another paradox illustrated for example by Schrödinger's cat (3, 9), but is able to resolve the EPR paradox because the system behaves deterministically as shall be demonstrated for a simple case in the following.

After the introduction of a discrete dynamical time in classical physics (3.1) as well as time resolution considerations of setting up and measuring an experiment (4.1), the super position of two quantum states is replaced by a fast harmonic oscillation between the states followed by measuring it classically under the assumption of a discrete time (4.2). In 4.3 the introduced concepts of discrete time and the fast oscillation between eigenstates are applied to the singlet state of two particles with spin $\frac{1}{2}$ without the request of entanglement resolving thereby the EPR paradox, followed in paragraph 5 by a discussion.

4 Theory

4.1 The discreteness of time in classical physics

It is assumed that in contrast to quantum mechanics with a continuous time t , in classical physics time evolves in very small discrete time steps Δt of constant nature (for example Δt could be the Planck time $\Delta t_p = \sqrt{\frac{hG}{2\pi c^3}} = 5.4 \cdot 10^{-44}$ s with h the Planck constant, c the light velocity and G the gravitational constant). This assumption is nourished on the finding that the arrow of time and entropy can be derived by the introduction of a discrete time [9] and on the request of a dual relationship between energy and time from the corresponding uncertainty principle $\Delta E \Delta t > h/2$ (i.e. since in quantum mechanics energy is quantized time is (allowed to be) continuous and vice versa, since in classical physics energy is continuous time is requested to be discrete). Thus, by introducing a discrete time, any time-dependent observable $A(t)$ is represented by a sequence of discrete values (10,11):

$$(A_0, t_0 = 0), (A_1, t_1 = 1 \Delta t), \dots, (A_n, t_n = n \Delta t), \dots, (A_{N+1}, t_{N+1} = [N + 1] \Delta t) \quad (1)$$

with (A_0, t_0) the initial and (A_{N+1}, t_{N+1}) the final measurement of the system under study and with n being an element of the natural numbers including 0. Furthermore, the following relationship between the continuous time in quantum mechanics t and the discrete time in classical physics holds:

$$t_n = n \Delta t \quad (2)$$

denoting t_n to be the quantum mechanical time at classical time point n . It is further assumed, that time is a single variable (unlike space coordinates) and started at the beginning of the universe and thus any object whether it is the system under study or the detector used (i.e. the instrument used to measure the system) are with the discrete time steps in tune/coherent to each other.

In addition, it is important to mention that with state of the art technologies the time precision of experiments and thus the time point of measurement as well as the start of the experiment are many orders of magnitude less accurate than Δt . Actually, since currently the time resolution is in the order of 10^{-18} s (or less) and assuming $\Delta t = \Delta t_p$, the starting time of the evolution of the system as well as its end are not more precise than ca. $10^{26} \Delta t$. Thus, each experiment is somewhat different and it is expected that only an ensemble averaging over many experiments will give a valuable result. With other words already because of these experimental limitations the time-dependent deterministic quantum mechanics described by the Schrödinger equation (3) can only in average calculate the outcome of an experiment. This argumentation suggests that quantum mechanics is well calculating a single outcome, but it is meaningless since the experimental set up is not of sufficient quality (i.e. of sufficient time resolution) and only upon ensemble averaging quantum mechanics and experiments fit to each other.

4.2 Modifying the super position of quantum states by a time-resolved fast oscillation between the states

Let us study a particle with spin $\frac{1}{2}$. In the standard description of quantum mechanics (3), the eigenstates of a spin $\frac{1}{2}$ along the z-axis are the spin up $|\uparrow\rangle_z$ state and the corresponding orthogonal spin down state $|\downarrow\rangle_z$. When the particle state is not defined (i.e. is not in one of the eigen states), it is said that the particle is in both states, it is super imposed. This can be described by the following wave function using the Dirac notation:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} |\downarrow\rangle_z + \frac{1}{\sqrt{2}} |\uparrow\rangle_z \quad (3)$$

While superimposed upon a single measurement, it is either in one of the two states, and upon ensemble averaging of many measurements it is 50% in the spin up state and 50% in the spin down state, expressed as follows:

$$\langle\Psi(t)|\Psi(t)\rangle_z = \frac{1}{2} \langle\downarrow|\downarrow\rangle_z + \frac{1}{2} \langle\uparrow|\uparrow\rangle_z \quad (4)$$

It is now proposed here to modify the super position concept by suggesting that the spin $\frac{1}{2}$ system oscillates between the two states harmonically (Figure 1) with either

$$|\Psi_{hd}^\downarrow(t)\rangle = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi t}{2\Delta t}\right) |\downarrow\rangle_z + \frac{1}{\sqrt{2}} \sin\left(\frac{\pi t}{2\Delta t}\right) |\uparrow\rangle_z \quad (5)$$

if at starting time the quantum state was in a spin down $|\downarrow\rangle_z$ state,

or, with

$$|\Psi_{hd}^\uparrow(t)\rangle = \frac{1}{\sqrt{2}} \sin\left(\frac{\pi t}{2\Delta t}\right) |\downarrow\rangle_z + \frac{1}{\sqrt{2}} \cos\left(\frac{\pi t}{2\Delta t}\right) |\uparrow\rangle_z \quad (6)$$

if at the start of the experiment the quantum system was in a spin up state $|\uparrow\rangle_z$. It is evident that with these wave function descriptions denoted $\Psi_{hd}(t)$ with *hd* for harmonic and discrete, there is no superposition because the system oscillates forth and back between the two states with a periodicity of $4\Delta t$.

As discussed above in measuring a single experiment one needs to consider the imprecision in the starting and measurement time. Thus, it is for a single measurement unknown whether the spin $\frac{1}{2}$ system starts with spin up $|\uparrow\rangle_z$ or spin down $|\downarrow\rangle_z$. Correspondingly, because with the current experimental technologies the current time resolution of the measurement is many orders of magnitude larger than the

steps of the discrete time the measurement is done at $t_n = n \Delta t$ with n being an even or odd number. An averaging over many experiments needs to take into account these limitations.

Under these considerations let us assume the experiment did start with the spin $\frac{1}{2}$ in the spin down state $|\downarrow\rangle_z$ measuring

$$\langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_{z,n} = \frac{1}{2} \cos^2\left(\frac{\pi n \Delta t}{2\Delta t}\right) \langle \downarrow | \downarrow \rangle_z + \frac{1}{2} \sin^2\left(\frac{\pi n \Delta t}{2\Delta t}\right) \langle \uparrow | \uparrow \rangle_z \quad (7)$$

(note the “bra” part may be regarded as the expectation value of the measurement device) which results by measuring at a time t_n with an even $n = 2m$ (with m being an element of the natural numbers including 0) to

$$\langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_{z,2m} = \frac{1}{2} \cos^2\left(\frac{\pi 2m \Delta t}{2\Delta t}\right) \langle \downarrow | \downarrow \rangle_z = \frac{1}{2} \langle \downarrow | \downarrow \rangle_z \quad (8)$$

and for a time with an odd $n = 2m + 1$ to

$$\langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_{z,2m+1} = \frac{1}{2} \sin^2\left(\frac{\pi (2m+1) \Delta t}{2\Delta t}\right) \langle \uparrow | \uparrow \rangle_z = \frac{1}{2} \langle \uparrow | \uparrow \rangle_z \quad (9)$$

Thus, the spin $\frac{1}{2}$ system that has started with a spin down state $|\downarrow\rangle$ is always detected as a spin up or a spin down state although by our modified quantum mechanical description there is an oscillation between the two states.

When ensemble averaged over many measurements with either a measurement at even or odd time steps

$$\langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_z = \frac{1}{2} \langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_{z,2m} + \frac{1}{2} \langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_{z,2m+1} = \frac{1}{2} \langle \downarrow | \downarrow \rangle_z + \frac{1}{2} \langle \uparrow | \uparrow \rangle_z \quad (10)$$

and thus 50% of the case the system is in spin up and in 50% of the measurements it is in the spin down state, respectively.

Correspondingly, if the spin $\frac{1}{2}$ starts with the spin up state $|\uparrow\rangle$ it is detected as

$$\langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_{z,n} = \frac{1}{2} \sin^2\left(\frac{\pi n \Delta t}{2\Delta t}\right) \langle \downarrow | \downarrow \rangle_z + \frac{1}{2} \cos^2\left(\frac{\pi n \Delta t}{2\Delta t}\right) \langle \uparrow | \uparrow \rangle_z \quad (11)$$

which results by measuring at a time t_n with an even $n = 2m$ (with m being an element of the natural numbers including 0) to

$$\langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_{z,2m} = \frac{1}{2} \cos^2\left(\frac{\pi 2m \Delta t}{2\Delta t}\right) \langle \uparrow | \uparrow \rangle_z = \frac{1}{2} \langle \uparrow | \uparrow \rangle_z \quad (12)$$

and for a time with an odd $n = 2m + 1$ to

$$\langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_{z, 2m+1} = \frac{1}{2} \sin^2\left(\frac{\pi(2m+1)\Delta t}{2\Delta t}\right) \langle \downarrow | \downarrow \rangle_z = \frac{1}{2} \langle \downarrow | \downarrow \rangle_z \quad (13)$$

and under the condition that the spin system started with a spin up state averaging over many measurements with either measurements at even or odd time steps results in

$$\langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_z = \frac{1}{2} \langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_{z, 2m} + \frac{1}{2} \langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_{z, 2m+1} = \frac{1}{2} \langle \uparrow | \uparrow \rangle_z + \frac{1}{2} \langle \downarrow | \downarrow \rangle_z \quad (14)$$

However, an ensemble measurement must take into account both the time variation-induced variation in the starting state (i.e. either spin up or spin down) as well as the measurement at even or odd time steps. By doing so the following expression is obtained

$$\begin{aligned} \langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_z &= \frac{1}{2} \langle \Psi_{hd}^\uparrow(t) | \Psi_{hd}^\uparrow(t) \rangle_z + \frac{1}{2} \langle \Psi_{hd}^\downarrow(t) | \Psi_{hd}^\downarrow(t) \rangle_z \\ &= \frac{1}{2} \langle \downarrow | \downarrow \rangle_z + \frac{1}{2} \langle \uparrow | \uparrow \rangle_z = \langle \Psi(t) | \Psi(t) \rangle_z \end{aligned} \quad (15)$$

As demonstrated in eq. 15. the same result as in the conventional quantum mechanics approach is obtained but without requesting a simultaneous superposition of the two states generating a modified quantum mechanics that is upon exact measurement deterministic, while under current experimental time accuracies predicts statistically the outcome of an experiment identical to standard quantum mechanics.

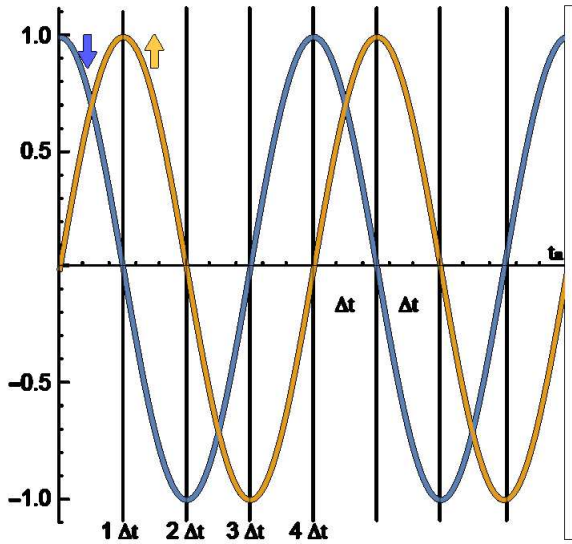


Figure 1: The time-resolved harmonic oscillation between spin down $|\downarrow\rangle_z$ and spin up $|\uparrow\rangle_z$ of a spin $\frac{1}{2}$ system is illustrated. With grey lines are the possible time points of the measurement indicated. They are in time steps of Δt as labeled.

4.3 The Einstein-Podolsky-Rosen paradox of a singlet state studied under discrete time physics

An entangled quantum system is discussed composed of two spins $\frac{1}{2}$ particles in a singlet state. This means that since the sum of the spin is 0, either of the particles is in the spin up state and the other in the spin down state, or vice versa, respectively. This singlet state can be described using the standard quantum mechanics formulation as follows:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle_z - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle_z \quad (16)$$

Please note, that because of the entanglement the wave function of the singlet state is not the product of the two individual ones $|\Psi_i(t)\rangle = \frac{1}{\sqrt{2}}|\downarrow_i\rangle_z + \frac{1}{\sqrt{2}}|\uparrow_i\rangle_z$ with $i = 1, 2$.

If the entangled state is made observable along the z-axis

$$\langle \Psi(t) | \Psi(t) \rangle_z = \frac{1}{2} \langle \uparrow\downarrow | \uparrow\downarrow \rangle_z + \frac{1}{2} \langle \downarrow\uparrow | \downarrow\uparrow \rangle_z = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \langle \downarrow_2 | \downarrow_2 \rangle_z + \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \langle \uparrow_2 | \uparrow_2 \rangle_z \quad (17)$$

which means that if particle 1 is upon measurement in the spin up state, particle 2 is simultaneously in the spin down state and vice versa. After many measurements, it is further the finding that in 50% of the cases particle 1 is in the spin up state and particle 2 is in the spin down state, while in the other 50% of measurements particle 1 is in the spin down state and particle 2 is in the spin up state, respectively.

For a more comprehensive analysis of the system under study, particle 1 is first detected along the z-axis followed by the measurement of particle 2 along an arbitrary angle φ to the z-axis [3, 4]. If the first measurement results in $\langle \uparrow_1 | \uparrow_1 \rangle_z$ the spin component of the wave function of particle 2 is given by $|\downarrow\rangle_z = \sin \frac{\varphi}{2} |\uparrow_2\rangle_\varphi + \cos \frac{\varphi}{2} |\downarrow_2\rangle_\varphi$. The probability that the result of the second experiment is also positive (i.e. $\langle \uparrow_2 | \uparrow_2 \rangle_\varphi$) is therefore given by $P_{++}(\varphi) = \sin^2 \frac{\varphi}{2}$ and correspondingly the probability that spin 2 is measured as a spin down state $\langle \downarrow_2 | \downarrow_2 \rangle_\varphi$ is $P_{+-}(\varphi) = \cos^2 \frac{\varphi}{2}$. (3). Accordingly, if the first measurement is $\langle \downarrow_1 | \downarrow_1 \rangle_z$ the probability that spin two is also negative is given by $P_{--}(\varphi) = \sin^2 \frac{\varphi}{2}$ and correspondingly the probability that spin 2 is measured as a spin up state is $P_{-+}(\varphi) = \cos^2 \frac{\varphi}{2}$, respectively (3).

In contrast to standard quantum mechanics described above, by using the harmonic oscillation and discrete time concept introduced above, the singlet state can be described by the product of the individual

states $|\Psi_{i,hd}(t)\rangle$ as follows:

$$|\Psi_{hd}(t)\rangle^{\downarrow\uparrow} = |\Psi_{1,hd}^{\downarrow}(t)\rangle * |\Psi_{2,hd}^{\uparrow}(t)\rangle \quad (18)$$

if particle 1 started with spin down and with

$$|\Psi_{hd}(t)\rangle^{\uparrow\downarrow} = |\Psi_{1,hd}^{\uparrow}(t)\rangle * |\Psi_{2,hd}^{\downarrow}(t)\rangle \quad (19)$$

if particle 1 started with spin up, respectively. Using the formulations of eqs. 5 and 6 these equations can be rewritten to

$$|\Psi_{hd}(t)\rangle^{\downarrow\uparrow} = \frac{1}{2} \left\{ \left[\cos\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z + \sin\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z \right] * \left[\sin\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_2\rangle_z + \cos\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_2\rangle_z \right] \right\} \quad (20)$$

$$\begin{aligned} &= \frac{1}{2} \cos^2\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z |\uparrow_2\rangle_z + \frac{1}{2} \sin^2\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z |\downarrow_2\rangle_z + \frac{1}{2} \cos\left(\frac{\pi t}{2\Delta t}\right) \sin\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z |\downarrow_2\rangle_z \\ &\quad + \frac{1}{2} \sin\left(\frac{\pi t}{2\Delta t}\right) \cos\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z |\uparrow_2\rangle_z \end{aligned}$$

and

$$|\Psi_{hd}(t)\rangle^{\uparrow\downarrow} = \frac{1}{2} \left\{ \left[\cos\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z + \sin\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z \right] * \left[\sin\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_2\rangle_z + \cos\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_2\rangle_z \right] \right\}$$

$$|\Psi_{hd}(t)\rangle^{\uparrow\downarrow} = \frac{1}{2} \left[\cos^2\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z |\downarrow_2\rangle_z + \frac{1}{2} \sin^2\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z |\uparrow_2\rangle_z + \frac{1}{2} \cos\left(\frac{\pi t}{2\Delta t}\right) \sin\left(\frac{\pi t}{2\Delta t}\right) |\uparrow_1\rangle_z |\uparrow_2\rangle_z \right. \quad (21)$$

$$\left. + \frac{1}{2} \sin\left(\frac{\pi t}{2\Delta t}\right) \cos\left(\frac{\pi t}{2\Delta t}\right) |\downarrow_1\rangle_z |\downarrow_2\rangle_z \right]$$

Please note, that the last terms with the mixed \sin and \cos functions are never observable (for any $t_n = n \Delta t$) because $\sin\left(\frac{\pi t}{2\Delta t}\right) \cos\left(\frac{\pi t}{2\Delta t}\right) = \frac{1}{2} \sin\left(\frac{\pi t}{\Delta t}\right)$. These terms although present are not detectable because they oscillate twice as fast and thus at any measurement time are 0. It is important to notice that at the origin of the doubling of the oscillation is the request for a single time variable: both particles are evolving with the same time and are thus connected to each other, while not entangled. Thus, if a measurement is made on particle 1 at a given time point it has also consequences for particle 2. For

example, if particle 1 started with spin down (i.e. eq. 20) and in the measurement is observed to have spin down $\langle \downarrow_1 | \downarrow_1 \rangle_z$ (i.e. $n = 2m$ with m being an element of the natural numbers including 0)

$$\langle \downarrow_1 | \Psi_{hd}(t) \rangle_{z,2m}^{\downarrow\uparrow} = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z | \uparrow_2 \rangle_z \quad (22)$$

for particle 2 the spin up state $\langle \uparrow_2 | \uparrow_2 \rangle_z$ will and must be detected and any time point later.

The same result is obtained if particle 1 started with spin up (eq. 21) and the measurement is performed at a time point t_n with $n = 2m + 1$

$$\langle \downarrow_1 | \Psi_{hd}(t) \rangle_{z,2m+1}^{\uparrow\downarrow} = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z | \uparrow_2 \rangle_z \quad (23)$$

Alternatively, if particle 1 is started with spin down (i.e. eq. 20) and the measurement of particle 1 is performed at a time point t_n with $n = 2m + 1$ the spin up state $\langle \uparrow_1 | \uparrow_1 \rangle_z$ is observed

$$\langle \uparrow_1 | \Psi_{hd}(t) \rangle_{z,2m+1}^{\uparrow\downarrow} = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z | \downarrow_2 \rangle_z \quad (24)$$

and thus for particle 2 the spin down state $\langle \downarrow_2 | \downarrow_2 \rangle_z$ will be detected at any time point later.

The same result is obtained if particle 1 is started with spin up (i.e. eq. 20) and the measurement of particle 1 is performed at a time point t_n with $n = 2m$ yielding the spin up state $\langle \uparrow_1 | \uparrow_1 \rangle_z$:
 $\langle \uparrow_1 | \Psi_{hd}(t) \rangle_{z,2m}^{\uparrow\downarrow} = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z | \downarrow_2 \rangle_z$

In analogy (i.e by exchanging the numbering of the spins in the formulas above) the corresponding results are obtained if first spin 2 is detected.

Finally, if averaged over many measurements in 50% of the measurements particle 1 is found in the spin up state and particle 2 in the down state, and vice versa in the other 50% of the measurements particle 1 is found in the spin down state and particle 2 in the spin up state, respectively:

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_z = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \langle \downarrow_2 | \downarrow_2 \rangle_z + \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \langle \uparrow_2 | \uparrow_2 \rangle_z = \langle \Psi(t) | \Psi(t) \rangle_z \quad (25)$$

Thus, under the assumption of a single coherent time variable, which is discrete in classical physics and by replacing the quantum super position by a fast oscillation between the states the detection along the z-axis of a singlet state can be described without the request of quantum entanglement. Actually, the quantum entanglement as exemplified here by the singlet state can be explained as two individual systems that are coherent in time and thus non-locally connected since it is assumed that there is only one single time variable.

This finding holds also if after the measurement of particle 1 along the z-axis at time point t_n , the second particle is measured at an angle φ to the z-axis at a later time point $t_{n+k} = t_n + k \Delta t$ (with k being an

element of the natural numbers including 0). It is important to notice that the request for a single time variable is thereby considered. Let us first study the case that particle 1 is measured at time point t_n to have spin down $\langle \downarrow_1 | \downarrow_1 \rangle_z$ (i.e. either eqs. 22 or 23). For preparing the measurement of particle 2 along the φ axis a time interval $\delta_k = k \Delta t$ later the following expression is obtained

$$\langle \downarrow_1 | \Psi_{hd}(t) \rangle_z = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \langle \uparrow_2 \rangle_z = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \left[\cos\left(\frac{\pi \delta_k}{2\Delta t}\right) \cos \frac{\varphi}{2} \langle \uparrow_2 \rangle_\varphi + \sin\left(\frac{\pi \delta_k}{2\Delta t}\right) \sin \frac{\varphi}{2} \langle \downarrow_2 \rangle_\varphi \right] \quad (26)$$

The measurement of particle 2 at a time interval $\delta_k = k \Delta t$ later with an even $k = 2 m'$ (with m' being an element of the natural numbers including 0).

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_{z,\varphi,2m'} = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \cos^2 \frac{\varphi}{2} \langle \uparrow_2 | \uparrow_2 \rangle_\varphi \quad (27)$$

and correspondingly the measurement at $\delta_k = k \Delta t$ later with an odd $k = 2 m' + 1$ yields

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_{z,\varphi,2m'+1} = \frac{1}{2} \langle \downarrow_1 | \downarrow_1 \rangle_z \sin^2 \frac{\varphi}{2} \langle \downarrow_2 | \downarrow_2 \rangle_\varphi \quad (28)$$

The corresponding results are obtained if the particle 1 is measured at time point t_n to have spin up $\langle \uparrow_1 | \uparrow_1 \rangle_z$ (i.e. eq. 24 and thereafter) and the measurement of particle 2 along the φ axis a time interval $\delta_k = k \Delta t$ later is prepared

$$\langle \uparrow_1 | \Psi_{hd}(t) \rangle_z = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \langle \downarrow_2 \rangle_z = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \left[\sin\left(\frac{\pi \delta_k}{2\Delta t}\right) \sin \frac{\varphi}{2} \langle \uparrow_2 \rangle_\varphi + \cos\left(\frac{\pi \delta_k}{2\Delta t}\right) \cos \frac{\varphi}{2} \langle \downarrow_2 \rangle_\varphi \right] \quad (29)$$

The measurement of particle 2 at $\delta_k = k \Delta t$ later with an $k = 2 m'$ yields

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_{z,\varphi,2m'} = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \cos^2 \frac{\varphi}{2} \langle \downarrow_2 | \downarrow_2 \rangle_\varphi \quad (30)$$

and correspondingly if measured at $\delta_k = k \Delta t$ later with $k = 2 m' + 1$ yields

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_{z,\varphi,2m'+1} = \frac{1}{2} \langle \uparrow_1 | \uparrow_1 \rangle_z \sin^2 \frac{\varphi}{2} \langle \uparrow_2 | \uparrow_2 \rangle_\varphi \quad (31)$$

When these individual measurements are ensemble averaged over many experiments the same probabilities are obtained as calculated by standard quantum mechanics:

$$\langle \Psi_{hd}(t) | \Psi_{hd}(t) \rangle_{z,\varphi} = \frac{1}{2} \cos^2 \frac{\varphi}{2} [\langle \uparrow_1 | \uparrow_1 \rangle_z \langle \downarrow_2 | \downarrow_2 \rangle_\varphi + \langle \downarrow_1 | \downarrow_1 \rangle_z \langle \uparrow_2 | \uparrow_2 \rangle_\varphi] \quad (32)$$

$$+\frac{1}{2}\sin^2 \frac{\varphi}{2}[\langle \uparrow_1 | \uparrow_1 \rangle_z \langle \uparrow_2 | \uparrow_2 \rangle_\varphi + \langle \downarrow_1 | \downarrow_1 \rangle_z \langle \downarrow_2 | \downarrow_2 \rangle_\varphi] = \langle \Psi(t) | \Psi(t) \rangle_{z,\varphi}$$

5 Discussion

In addition to a quantized energy, quantum mechanics differs from classical physics by several counter intuitive properties such as the super position of quantum states, the non local nature of quantum mechanics, and the lack of the possibility to calculate a single measurement. The presented approach of combining the presumed existence of a discrete time in classical physics coherently connected to a single continuous time variable in quantum mechanics and the fast oscillation of the quantum system between its eigenstates is able to resolve some of these phenomena. It localizes the problem of measuring a quantum state (i.e. collapse of the wave function) to the issue of exact timing and explains why quantum mechanics is not able to calculate single experimental outcomes at current time resolutions, but calculates them correctly in average. It further resolves the causality issue as outlined in the EPR paradox by Einstein and coworkers (1). In this context it is noted that a Gedankenexperiment by A. Suarez further shows that while standard quantum mechanics is non local, it is only so by requesting relativity, which is local in nature, a dilemma which can be resolved by the introduction of a discrete time (11). At the root of the causal nature of the presented modified quantum mechanics theory is thereby in addition the request of a single time variable, which is believed to be a logic axiom for a theory that guarantees causality.

While the presented modification of quantum mechanics for a two spin system may be regarded sound, the generalization - how it can be translated into a system with many eigen states - remains to be established. One may speculate that the system goes periodically through all the eigen states (please note, the mathematical description of such a system may include step functions, which may result in other demanding mathematical reformulations such as the Schrödinger equation).

Furthermore, finding experimental support for the presence of a discrete time and the modified quantum mechanics appear to be difficult since the current experimental time resolution is ca 26 orders of magnitude away from the Planck time Δt_p . Nonetheless, it can be stated, that the better the time resolution of the experiment (and eventually also the faster the processes under study) the more deviation from quantum mechanics towards a classical/deterministic behavior of the system is expected. Alternatively, the nature of the break down of a quantum mechanical system into its classical analog may give valuable hints in favor or against the presented theory. For example if a quantum system oscillates through all the eigen states in steps of $4 \Delta t_p$ as speculated above, the system may behave classically or/and deterministic if the time the system needs to go through all significant eigen states is longer than the experimental time resolution (i.e. if $f * 4 \Delta t_p$ is larger than the time resolution of the experiment with f being the number of significant eigen states of the system), while it would behave quantum mechanically if the time resolution of the experiment would be less (please note at the current time resolution this requests the study of a system with ca 10^{26} significantly populated eigen states). With other words, if time is discrete the

presented theory is able to calculate the size dependence of a system to behave as either a quantum or classical system.

It is evident that the concept of a discrete time is at the root of the presented interpretation. It is however not a very popular concept, although upon energy quantization in quantum mechanics the introduction of a discrete time appears to be obvious (12-17). Of course, in the presented manuscript, only the simplest examples have been looked at yet and further considerations are necessary to show generality (see also discussion above). However the recent success in the derivation of a microscopic entropy under the assumption of a discrete time strengthens the hypothesis that time in classical physics may be discrete (10) - a possibility that is worth to be disputed further.

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7 References

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